

General Certificate of Education  
June 2008  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Further Pure 1**

**MFP1**

Monday 16 June 2008 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 4 and 8 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 The equation

$$x^2 + x + 5 = 0$$

has roots  $\alpha$  and  $\beta$ .

- (a) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . (2 marks)
- (b) Find the value of  $\alpha^2 + \beta^2$ . (2 marks)
- (c) Show that  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{9}{5}$ . (2 marks)
- (d) Find a quadratic equation, with integer coefficients, which has roots  $\frac{\alpha}{\beta}$  and  $\frac{\beta}{\alpha}$ . (2 marks)

2 It is given that  $z = x + iy$ , where  $x$  and  $y$  are real numbers.

- (a) Find, in terms of  $x$  and  $y$ , the real and imaginary parts of

$$3iz + 2z^*$$

where  $z^*$  is the complex conjugate of  $z$ . (3 marks)

- (b) Find the complex number  $z$  such that

$$3iz + 2z^* = 7 + 8i$$
 (3 marks)

3 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a)  $\int_9^{\infty} \frac{1}{\sqrt{x}} dx$ ; (3 marks)

(b)  $\int_9^{\infty} \frac{1}{x\sqrt{x}} dx$ . (4 marks)

4 [Figure 1 and Figure 2, printed on the insert, are provided for use in this question.]

The variables  $x$  and  $y$  are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where  $a$  and  $b$  are constants.

(a) The variables  $X$  and  $Y$  are defined by  $X = x(x+2)$ ,  $Y = y(x+2)$ .

Show that  $Y = aX + b$ .

(2 marks)

(b) The following approximate values of  $x$  and  $y$  have been found:

$x$	1	2	3	4
$y$	0.40	1.43	2.40	3.35

(i) Complete the table in **Figure 1**, showing values of  $X$  and  $Y$ .

(2 marks)

(ii) Draw on **Figure 2** a linear graph relating  $X$  and  $Y$ .

(2 marks)

(iii) Estimate the values of  $a$  and  $b$ .

(3 marks)

5 (a) Find, in **radians**, the general solution of the equation

$$\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of  $\pi$ .

(5 marks)

(b) Hence find the smallest **positive** value of  $x$  which satisfies this equation.

(2 marks)

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) Calculate the matrix **AB**.

(2 marks)

(b) Show that  $\mathbf{A}^2$  is of the form  $k\mathbf{I}$ , where  $k$  is an integer and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix.

(2 marks)

(c) Show that  $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$ .

(3 marks)

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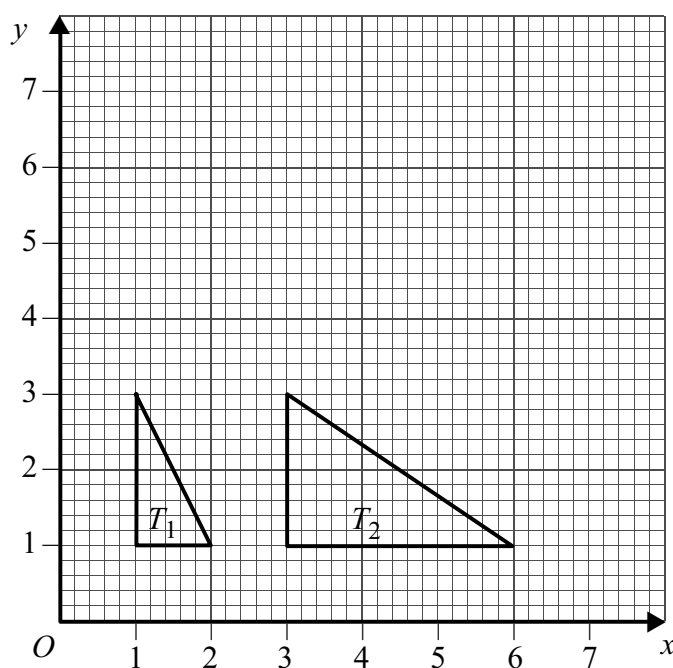
7 A curve  $C$  has equation

$$y = 7 + \frac{1}{x+1}$$

- (a) Define the translation which transforms the curve with equation  $y = \frac{1}{x}$  onto the curve  $C$ . (2 marks)
- (b) (i) Write down the equations of the two asymptotes of  $C$ . (2 marks)
- (ii) Find the coordinates of the points where the curve  $C$  intersects the coordinate axes. (3 marks)
- (c) Sketch the curve  $C$  and its two asymptotes. (3 marks)

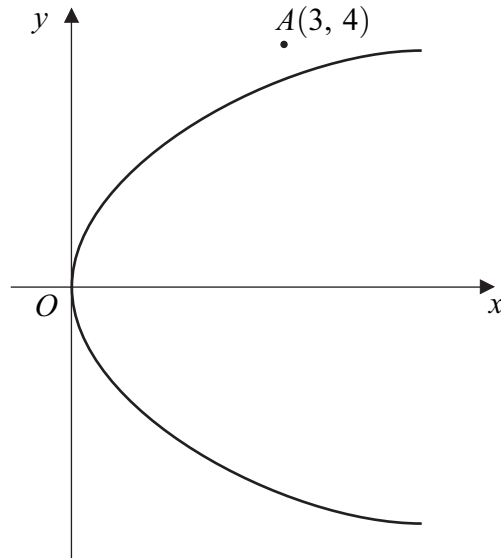
8 [Figure 3, printed on the insert, is provided for use in this question.]

The diagram shows two triangles,  $T_1$  and  $T_2$ .



- (a) Find the matrix of the stretch which maps  $T_1$  to  $T_2$ . (2 marks)
- (b) The triangle  $T_2$  is reflected in the line  $y = x$  to give a third triangle,  $T_3$ .  
 On **Figure 3**, draw the triangle  $T_3$ . (2 marks)
- (c) Find the matrix of the transformation which maps  $T_1$  to  $T_3$ . (3 marks)

9 The diagram shows the parabola  $y^2 = 4x$  and the point  $A$  with coordinates  $(3, 4)$ .



(a) Find an equation of the straight line having gradient  $m$  and passing through the point  $A(3, 4)$ . (2 marks)

(b) Show that, if this straight line intersects the parabola, then the  $y$ -coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0 \quad (3 \text{ marks})$$

(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through  $A$ .

(No credit will be given for solutions based on differentiation.) (5 marks)

(d) Find the coordinates of the points at which these tangents touch the parabola. (4 marks)

**END OF QUESTIONS**

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Centre Number						Candidate Number					
Candidate Signature											

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# Insert

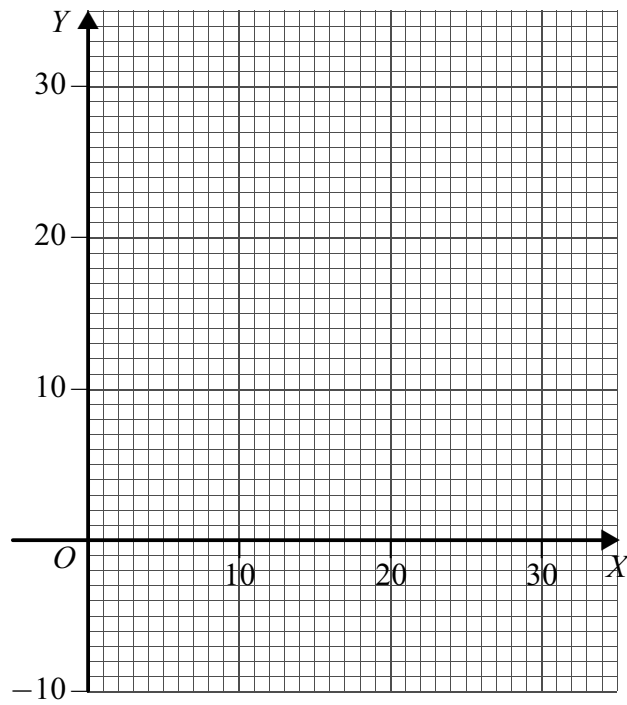
Insert for use in **Questions 4 and 8**.  
Fill in the boxes at the top of this page.  
Fasten this insert securely to your answer book.

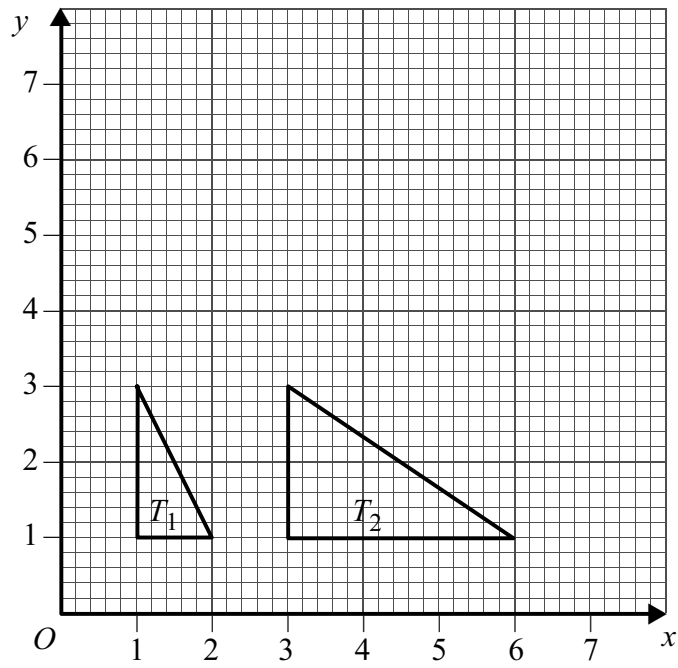
**Turn over for Figure 1**

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**Figure 1 (for use in Question 4)**

$x$	1	2	3	4
$y$	0.40	1.43	2.40	3.35
$X$	3			
$Y$	1.20			

**Figure 2 (for use in Question 4)**

**Figure 3 (for use in Question 8)**

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